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Paper

Building a Type-2 Fuzzy Regression Model based on Creditability Theory and its application on Arbitrage Pricing Theory

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Type-2 (T2) fuzzy set was introduced to model vagueness associated with primary membership function of type-1(T1) fuzzy set. While it was invented to handle more fuzziness of information, there are only a few algorithms(models) to deal with data in the form of T2 fuzzy variables given their three-dimensional features. To solve the problem, we define the expected value of a T2 fuzzy variable using creditability theory in this paper. And by substituting the expected value for the original T2 fuzzy set , the vertical uncertainties of data are transferred to horizontal ones without much distortion of information. Calculations between three dimensional T2 fuzzy sets are thus transferred to two-dimensional range calculations between T1 fuzzy sets. Based on that principle, we also build a T2 fuzzy expected regression model and apply it on the arbitrage pricing theory.

Keywords: T2 fuzzy set, regression model, creditability theory, expected value

1. Introduction

Information in real life may have linguistically vagueness. Traditional set theory that uses characteristic function to define whether an element belongs to a certain set (event) doesn't concern such uncertainty. Fuzzy set (T1 fuzzy set) was first introduced for the problem in 1965 by Lofti A Zadeh [1]. After that, Watada and Tanaka expanded a fuzzy quantification method in 1987 [2]. From then on, it is able to describe an artificial membership function with its output called primary membership grades, to which extend one element belongs to a certain set (event).

On the background that the membership function of a T1 fuzzy set may also have uncertainty associated with it, Lotfi A. Zadeh invented Type-2 fuzzy sets(T2 fuzzy variable) in 1975 [3]. A T2 fuzzy set lets us incorporate fuzziness about the membership function into fuzzy set theory and is a way to address the above concern of T1 fuzzy sets head-on. However, T2 fuzzy set didn't become popular immediately given its complexity of calculation. T2 fuzzy sets are difficult to understand because: (1) T2 fuzzy sets are more theoretical, thus less of empirical applications. (2) using T2 fuzzy sets is computationally more complicated than using T1 fuzzy sets. The conception was only investigated by a few researchers; for example, Mizumoto and Tanaka [4] discussed what kinds of algebraic structures the grades of T2 fuzzy sets form under join, meet and negation; Dubois and Prade [5] investigated the operations in a fuzzy-valued logic. It is not until recent days that T2 fuzzy sets have been applied successfully to T2 fuzzy logic systems to handle linguistic and numerical uncertainties [6][7][8][9][10].

On the other hand, various fuzzy regression models were introduced to cope with qualitative data coming from fuzzy environments where human (expert) subjective estimates are used. The first fuzzy linear regression model was proposed by Tanaka[11]. Tanaka[12], Tanaka and Watada[13], Watada and Tanaka[14] presented possibilistic regression based on the concept of possibility measure. Chang[15] discussed a fuzzy least-squares regression, by using weighted fuzzy-arithmetic and the least-squares fitting criterion. Watada[16] developed models of fuzzy time-series by exploiting the concept of intersection of fuzzy numbers.

For the reason mentioned above, most of the existing studies on fuzzy logic system(called "FLC")have focused on data consisting of numeric values or T1 fuzzy variables without T2 hybrid uncertainty into consideration. While in practical situations, there exists a growing need to cope with data in presence of more complicated uncertainty including fields such as industrial control[17][18][19][20], pattern recognition[21][22][23], decision making technology[24][25], healthcare[26][27], financial engineering[28][29][30] and communication systems[31][32][33]. In these applications, T2 fuzzy set either models complex uncertainties which T1 fuzzy set cannot do or constructs a robust control/prediction system that outperformed the traditional T1 FLCs. As the type-2 fuzzy set membership functions are themselves fuzzy and contain a footprint of uncertainty, they can model and handle the linguistic and numerical uncertainties associated with the inputs and outputs of the FLC in changing and dynamic unstructured environments and hence they can handle the difficulty associated with determining the exact membership functions for the fuzzy sets. Therefore, FLCs that are based on type-2 fuzzy sets will have the potential to produce a better performance than type-1 FLCs when dealing with uncertainties. For example, traditionally T1 Fuzzy PID controllers have been used for speed control in marine/traction diesel engines. However, there are many sources of uncertainty which is essential facing the FLC for marine/traction diesel engines

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that cannot be modeled by T1F set in practice, such as uncertainties in inputs to the FLC which translate to uncertainties in the antecedents membership functions as sensors measurements are affected by high noise levels from various sources like electromagnetic and radio frequency interference, vibration induced triboelectric cable charges, etc; Uncertainties in control outputs which translate to uncertainties in the output membership functions of the FLC. Such uncertainties can result from change of the actuators characteristics due to wear and tear or environmental changes such as in altitude which have direct effect on combustion; Uncertainties in the engine operation conditions which can be due to change of load, change of fuel, etc. Such uncertainties can translate to uncertainties in the antecedents and/or consequents membership functions. All of these uncertainties translate into uncertainties about fuzzy set membership functions, thus T2 fuzzy set is more proper to model it.

Another example is in financial market. There are two types of prices in financial markets. One is called transaction price and the other is offered price. The former is always continuous and objective based on real transactions, while the latter is discrete and subjective based on quotations. The price for Libor (London Inter-Bank Offered Rate)is an offered price. The majority investment banks offered quotations for borrowing/lending rate of money in inter-bank market according to their estimation of the future trend of financial markets. The market regulator collects all those quotations, adds some weight for each bank, then calculates an average result as the current Libor. The Libor contains all majority banks' opinions of the market and will be the basis for valuations of all other financial assets. Moreover, regulators will judge the quotation performance of each bank by some standards(called evaluation system) and give the evaluation to each quoter from time to time. The evaluation will restrict behaviors of quoters meanwhile as a standard to adjust their weight percentage to final result. T here are only a few models applying offered prices as inputs to make estimations or predictions because the price is hard to quantify for its two layer uncertainties, which are the quotation range and regulators evaluation. We use primary membership grade to describe how the quotation belonging to the to-dates price and use secondary membership grade to model those effectiveness on regulators evaluations.

However, with regard to the complexity of type-2 fuzzy variables, there are only a few mathematical algorithms modeling, learning T2 fuzzy inputs and predicting T2 fuzzy outputs. Recently, Wei and Watada developed T2F qualitative regressions model[34][35][36]. However, the model only applies type-2 fuzzy variables as coefficients of system but inputs and outputs. Thank to Prof Liu[37][38], he created a notion of credibility measure for fuzzy sets, which is a convex combination of possibility measure and necessity measure. By using it , we are able to make the calculation of fuzzy sets much easier than before. We obtained the idea to build an advanced T2 fuzzy regression algorithm.

The objective of this paper is to introduce a class of T2 fuzzy expected regression model based on creditability theory to deal with T2 fuzzy inputs and outputs. We use creditability theory introduced by Liu to define the expected value of a T2 fuzzy variable. Given that there is few applications for T2 fuzzy sets, we also apply the T2 fuzzy regression model to the arbitrage pricing theory and compare its prediction result with T1 fuzzy regression model and traditional regression model. This paper will be a further work based on our former research of type-2 fuzzy qualitative regression model.

The remainder of this paper is organized as follows. In Section 2, we cover some preliminaries of creditability theory and T2 fuzzy sets. Then we define the expected value of T2 fuzzy variable in section 3. Section 4 formulates a T2 fuzzy expected regression model. In section 5, a heuristic algorithm will be offered to solve the problem. Moreover, we apply the new model to arbitrage pricing theory in section 7. Finally, concluding remarks are presented in Section 8.

2. Preliminaries

2.1 Creditability Theory Recently, Liu has succeeded in establishing an axiomatic foundation for uncertainty. He created a notion of credibility measure, which is a convex combination of possibility measure and necessity measure. Given some universe Γ, let Pos be a possibility measure defined on the power set $\mathcal{P}(\Gamma)$ of Γ. Let $\Re$ be the set of real numbers. A function $A : \Gamma \to \Re$ is said to be a fuzzy variable defined on Γ . The possibility distribution μ_A of A is defined by $\mu_A(t) = \mathrm{Pos}\{A = t\}$, $t \in \Re$, which is the possibility of event $\{A = t\}$. For fuzzy variable A with possibility distribution μ_A, the possibility, necessity and credibility of event $\{A \le r\}$ are given, as follows

$$\begin{aligned}\mathrm{Pos}\{A \le r\} &= \sup_{t\le r}\mu_A(t),\\ \mathrm{Nec}\{A \le r\} &= 1 - \sup_{t>r}\mu_A(t),\\ \mathrm{Cr}\{A \le r\} &= \frac{1}{2}\left(1 + \sup_{t\le r}\mu_A(t) - \sup_{t>r}\mu_A(t)\right).\end{aligned} \quad \cdots\cdots (1)$$

From (1), we note that the credibility measure is an average of the possibility and the necessity measure, i.e., $\mathrm{Cr}\{\cdot\} = (\mathrm{Pos}\{\cdot\} + \mathrm{Nec}\{\cdot\})/2$, and it is a self-dual set function, i.e., $\mathrm{Cr}\{A\} = 1 - \mathrm{Cr}\{A^c\}$ for any A in $\mathcal{P}(\Gamma)$. The credibility theory is first introduced by Liu.The motivation behind the introduction of the credibility measure is to develop a certain measure which is a sound aggregate of the two extreme cases such as the possibility (expressing a level of overlap and being highly optimistic in this sense) and necessity (articulating a degree of inclusion and being pessimistic in its nature). Moreover, we are able to calculate the expected value of a fuzzy set from then on. For fuzzy variables, there are many ways to define an expected value operator. See, for example,Dubois and Prade , Heilpern, Campos and Gonzalez, Gonzalez and Yager. The most general definition of expected value operator of fuzzy variable was given by Liu. Based on credibility measure, the expected value of a fuzzy variable is presented as follows.

Let A be a T1 fuzzy variable. The expected value of A is defined as

$$E[A] = \int_0^\infty \mathrm{Cr}\{A \ge r\}\mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\{A \le r\}\mathrm{d}r \cdots\cdots (2)$$

provided that the two integrals are finite.

Moreover, assume that $A = (a, c^l, c^r)_T$ is a triangular fuzzy variable whose possibility distribution is

$$\mu_A(x) = \begin{cases} \frac{x - c^l}{a - c^l}, & c^l \leq x \leq a \\ \frac{c^r - x}{c^r - a}, & a \leq x \leq c^r \\ 0, & \text{otherwise.} \end{cases} \cdots\cdots\cdots\cdots\cdots (3)$$

Making use of (2), we determine the expected value of A to be

$$E[A] = \frac{c^l + 2a + c^r}{4}. \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

2.2 Type-2 Fuzzy Set Developed from T1 fuzzy sets, T2 fuzzy sets express the non-numeric membership with imprecision and uncertainty. A T2 fuzzy set denoted by $\widetilde{A}$, is characterized by a T2 membership function $\mu_{\widetilde{A}}(x, \mu_A(x))$, where $x \in X$ and $\mu_A(x) \in J_x \subseteq [0, 1]$. The elements of the domain of $\mu_A(x)$ are called primary memberships of x in A and the memberships of the primary memberships in $\mu_{\widetilde{A}}(x)$ are called secondary memberships of x in $\widetilde{A}$. i.e.,

$$A = \{x, \mu_A(x) | x \in X\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

$$\widetilde{A} = \{(x, \mu_A(x)), \mu_{\widetilde{A}}(x, \mu_A(x)) | \\ x \in X \ \mu_A(x) \in J_x \subseteq [0, 1]\} \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

in which $\mu_A(x) \in J_x \subseteq [0, 1]$ and $\mu_{\widetilde{A}} \subseteq [0, 1]$. T2 fuzzy variable $\widetilde{A}$ can also be expressed as

$$\int_{(x \in X)} \int_{(\mu_A(x) \in J_x \subseteq [0,1])} \mu_{\widetilde{A}}(x, \mu_A(x))/(x, \mu_A) \quad \cdots\cdots (7)$$

Another important concept with regard to T2 fuzzy set is the footprint of uncertainty. Uncertainty in the primary memberships of a type-2 fuzzy set, consists of a bounded region that we call the footprint of uncertainty (FOU). It is the union of all primary memberships, i.e.,

$$FOU(\widetilde{A}) = \bigcup_{x \in X} J_x \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

The term footprint of uncertainty is useful, because it not only focuses our attention on the certainties inherent in a specific T2 fuzzy membership function, whose shape is a direct consequence of the nature of these uncertainties, but it also provides a convenient verbal description of the entire domain of support for all the secondary grades of a T2 membership function. It also enables us to depict a T2 fuzzy set graphically in two-dimensions instead of three dimensions, and by doing so lets us overcome the first difficulty about T2 fuzzy sets-their three-dimensional nature which makes them very difficult to draw. The shaded FOUs imply that there is a distribution that sits on top of the new third dimension of T2 fuzzy sets. What that distribution looks like depends on the specific choice made for the secondary grades. When they all equal one, the resulting type-2 fuzzy sets are called interval T2 fuzzy sets. Such sets are the most widely used T2 fuzzy sets to date.

2.3 Type-2 Fuzzy Logic System The conventional fuzzy rule-base structures employ type-1 fuzzy sets as inputs, outputs and both/either in antecedent and/or consequent parts of the rules. However, the uncertainty can be captured in a better way by using higher order fuzzy sets, such as Type-2 fuzzy sets, which encapsulate more information granules. Figure 1 shows the structure of a type-2 fuzzy logic system(FLS). For a type-2 FLS, the inputs and outputs are type- 2 fuzzy sets and the same is to both/either in antecedent and/or consequent parts of the rules. It has been argued that using type-2 fuzzy sets to represent the inputs and outputs of FLCs has many advantages when compared to type-1 fuzzy sets; We summarize some of these advantages as follows£

1¢As the type-2 fuzzy set membership functions are themselves fuzzy and contain a footprint of uncertainty, they can model and handle the linguistic and numerical uncertainties associated with the inputs and outputs of the FLC in changing and dynamic unstructured environments and hence they can handle the difficulty associated with determining the exact membership functions for the fuzzy sets. Therefore, FLCs that are based on type-2 fuzzy sets will have the potential to produce a better performance than type-1 FLCs when dealing with uncertainties.

2¢Using type-2 fuzzy sets to represent the FLC inputs and outputs will result in the reduction of the FLC rule base when compared to using type-1 fuzzy sets as the uncertainty represented in the footprint of uncertainty in type-2 fuzzy sets lets us cover the same range as type-1 fuzzy sets with a smaller number of labels. The rule reduction will be greater as the number of the FLC inputs increases.

3¢Each input and output will be represented by a large number of type-1 fuzzy sets which are embedded in the type-2 fuzzy sets. The use of such a large number of type-1 fuzzy sets to describe the input and output variables allows for a detailed description of the analytical control surface as the addition of the extra levels of classification gives a much smoother control surface and response.

According to Karnik and Mendel, the type-2 FLC can be thought of as a collection of many different embedded type-1 FLCS. It has been proved that the extra degrees of freedom provided by the footprint of uncertainty enables a type-2 FLC to produce outputs that cannot be achieved by type-1 FLCs with the same number of membership functions. It has also been shown that a type-2 fuzzy set may give rise to an equivalent type-1 membership grade that is negative or larger than unity. Thus a type-2 FLC is able to model more complex input-output relationships than its type-1 counterpart and thus can give a better control response. There are many evidences in practical applications:

Christopher Lynch, Hani Hagras, Victor Callaghan[18] introduce a real-time type-2 FLC for embedded controllers operating in marine/traction diesel engines and conducted numerous experiments where the embedded type-2 FLCs dealt with the uncertainties in real -time and displayed a robust control response that outperformed the PID and type-1 FLCs whilst using smaller rule bases.

Jia Zeng, Zhi-Qiang Liu[23] presents an extension of hidden Markov models (HMMs) based on the type-2 (T2) fuzzy set (FS) referred to as type-2 fuzzy HMMs (T2 FHMMs), then apply them to phoneme classification and recognition on the TIMIT speech database. Experimental results show that T2 FHMMs can effectively handle noise and dialect uncertainties in speech signals besides a better classification performance than the classical HMMs.

Qilian Liang and Lingming Wang[39] present a new approach for sensed signal strength forecasting in wireless sensors using interval type-2 fuzzy logic system (FLS) and compare the interval type-2 FLS or sensed signal forecasting against a type-1 FLS. Simulation results show that the interval type-2 FLS performs much better than the type-1 FLS in sensed signal forecasting.

M.H. Fazel Zarandi a E. Neshata [40] developed a type-2 Fuzzy Rule Based Expert System to analyze the stock markets. The fuzzy rule based model is tested on the stock market of an automotive manufactory in Asia and the prediction result is much better then type-1 FLC.

3. Expected Value of T2 fuzzy set based on Creditability Theory

After introducing creditability theory and the conception of T2 fuzzy sets, we define the expected value of T2 fuzzy set using creditability measure here. We see later that using the expected value to model T2 fuzzy variables reduces huge complexity in the process of calculation, transfer the vertical uncertainty into horizontal one , thus enable us to deal with T2 fuzzy variables.

Suppose that $(\Gamma_1, \mathcal{P}(\Gamma_1), \mathrm{Pos}_1)$ is a possibility space. Let Γ_1 be the universe of discourse, and $\mathcal{P}(\Gamma_1)$ on Γ_1 is a class of subsets of Γ_1 that is closed under arbitrary unions, intersections, and complement in Γ_1. Let $\Re$ be the set of real numbers. Then a map $\mu_A : \Gamma_1 \to \Re$ is said to be a T1 fuzzy membership function defined on Γ_1. We also define another possibility space for T2 fuzzy sets, which is $(\Gamma_2, \mathcal{P}(\Gamma_2), \mathrm{Pos}_2)$. A function $\mathrm{Pos}_2 : \mathcal{P}(\Gamma_2) \to [0, 1]$ and a T2 fuzzy membership function is a mapping $\mu_{\widetilde{A}} : \Gamma_2 \to \Gamma_1$. The most normal T2 fuzzy sets are constructed in an interval style which is illustrated as follows:

Let A be a fuzzy set defined on possibility space $(\Gamma_1, \mathcal{P}(\Gamma_1), \mathrm{Pos})$. Define that for every $\mu_A \in \Gamma_1$, $\mu_{\widetilde{A}} = (\mu_A + c^l, \mu_A + c^r)\Gamma_2$, which is a T2 interval fuzzy membership function defined on some possibility space $(\Gamma_2, \mathcal{P}(\Gamma_2), \mathrm{Pos})$.

Moreover, a type-2 fuzzy variable is defined as a mapping $\widetilde{A}_v : \Gamma_2 \to \Re$. The normal form of a T2 fuzzy variable is in the form of interval and a more complicated one is in the form of triangle defined as follows:

Let A be a fuzzy variable defined on possibility space $(\Gamma_1, \mathcal{P}(\Gamma_1), \mathrm{Pos})$. For every $x \in \Re$, it follows a triangular primary membership function as $A(x) = (x+a, x+c^l, x+c^r)$ and for every $\mu_A \in \Gamma_1$, it also follows a triangular secondary membership function $\widetilde{A}(\mu_A) = (\mu_A(x_0), \mu_A(x_0)+e^l, \mu_A(x_0)+e^r)$. For every $\mu_{\widetilde{A}} \in \Gamma_1$.

For any T2 fuzzy set $\widetilde{A}$, the expected value of the T2 fuzzy set $\widetilde{A}$ is denoted by $E[\mu_{\widetilde{A}}]$ given μ_A, which has been proved to be a measurable function of A i.e., it is a T1 fuzzy variable. The mathmatical definition is as follows:

Let $\widetilde{A}_v$ be a T2 fuzzy variable and $\mu_{\widetilde{A}}$ be a T2 fuzzy membership function defined on a possibility space $(\Omega, P(\Omega), \mathrm{Pos})$, which describes the secondary membership grades for$\widetilde{A}_v$. The expected value of T2 fuzzy variable $\widetilde{A}_v$ is defined as follows.

$$E[\xi] = \int_\Omega \left[\int_0^\infty Cr\{\xi(\omega) \geq r\}dr - \int_{-\infty}^0 Cr\{\xi(\omega) \leq r\}dr \right] Pr(d\omega)$$

(9)

Assume that original outputs for the primary membership function of $\widetilde{A}_v$ is the following possibility distribution of $\mu_A(x_0)$, where $x_0 \in \Re$ In order to get the expected value of T2 fuzzy variable, we may take the place of the original primary grades by using the result of equation above. Hence, μ_A will be transformed into a new function defined as expected primary membership function denoted as $\mu_{E[\widetilde{A}]}$. And its creditability is denoted as $\mathrm{Cr}_{E[\widetilde{A}]}$ instead of Cr_A.

We give a simple example here to help understand the definition. Assume that there is a triangular T2 fuzzy variable $\widetilde{A}_0$. Its primary membership function for real values is $A = (a, c^l, c^r)\Gamma_1$ whose possibility distribution is

$$\mu_A(x) = \begin{cases} \dfrac{x - c^l}{a - c^l}, & c^l \leq x \leq a \\ \dfrac{c^r - x}{c^r - a}, & a \leq x \leq c^r \\ 0, & \text{otherwise.} \end{cases} \quad \cdots\cdots\cdots\cdots\cdots (10)$$

Meanwhile, for any $x_0 \in \Re$ included in $A = (a, c^l, c^r)\Gamma_1$ with its primary grades expressed as $\mu_A(x_0)$, the T2F membership function for $\mu_A(x_0)$ is assumed to be $\widetilde{A}(\mu_A(x_0)) = (\mu_A(x_0), \mu_A(x_0) + e^l, \mu_A(x_0) + e^r)\Gamma_2$, whose possibility distribution is

$$\mu_{\widetilde{A}}(\mu_A(x)) = \begin{cases} \dfrac{\mu_A(x) - e^l}{\mu_A(x_0) - e^l}, & e^l \leq \mu_A(x) \leq \mu_A(x_0) \\ \dfrac{e^r - \mu_A(x)}{e^r - \mu_A(x_0)}, & \mu_A(x_0) \leq \mu_A(x) \leq e^r \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

where $A \in \Gamma_1, \widetilde{A} \in \Gamma_2$. Notice that $\mu_A(x_0)$ will be the center of the T2 fuzzy membership grades.

For boundaries of primary grades of $\widetilde{A}_0$ where values of them are 0, the secondary membership function is $\widetilde{A}(0) = (0, 0 + e^l, 0 + e^r)\Gamma_2$, and the expected value of them according to equation (7) will be

$$E[A] = \frac{e^l + e^r}{4}.$$

For center of primary grades where its value is 1, the secondary membership function is $\widetilde{A}(b) = (1, 1 + e^l, 1 + e^r)\Gamma_2$, and the expected value will be

$$E[A] = \frac{e^l + 2 + e^r}{4}$$

. One thing that should be noticed is that even there will be surplus more than 1 in the calculation process, it will not affect the final result. Then we substitute expected primary grades for original ones and form an expected primary membership function. The original one satisfies $\mu_A(a) = 1$, $\mu_A(c^l) = 0$ and $\mu_A(c^r) = 0$. After transformation, the expected one follows distribution as the following equation and satisfies $\mu_A(a) = \frac{e^l+2b+e^r}{4}$, $\mu_A(c^l) = \frac{e^l+e^r}{4}$ and $\mu_A(c^r) = \frac{e^l+e^r}{4}$.

$$\mu_{E[\widetilde{A}]}(x) = \begin{cases} \dfrac{x - c^l}{2(a - c^l)} + \dfrac{e^l + e^r}{4}, & c^l \leq x \leq a \\ \dfrac{c^r - x}{2(c^r - a)} + \dfrac{e^l + e^r}{4}, & a \leq x \leq c^r \\ \frac{e^l+e^r}{4}, & \text{otherwise.} \end{cases} \quad \cdot (12)$$

Noticed that after transformation into the expected primary function, the function biases a value at $\frac{e^l+e^r}{4}$ higher for the universe, which is not reasonable. Thus we need to make some adjustment through shaping the function into triangular one again. Actually, after transform the T2F set into T1F set, the new boundaries are loosing thus larger than original ones. It is because the new reduced T1F set contains more information than the original T1F set. We may calculate the new expected value of $\widetilde{A}_0$ is $A' = (c^l - \frac{(e^r+e^l)(a-c^l)}{2}, c^r + \frac{(e^r+e^l)(c^r-a)}{2})\Gamma_1$ instead of $A = (c^l, c^r)\Gamma_1$, whose possibility distribution follows:

$$\mu_{E[\widetilde{A}]}(x) = \begin{cases} \dfrac{x - c^l}{2(a - c^l)}, & c^l - \frac{(e^r+e^l)(a-c^l)}{2} \le x \le a \\ \dfrac{c^r - x}{2(c^r - a)}, & a \le x \le a \le x \le c^r + \frac{(e^r+e^l)(c^r-a)}{2} \\ 0, & \text{otherwise.} \end{cases}$$

the creditability of the expected primary function will be like:

$$\begin{aligned} \Phi(x) &= \mathrm{Cr}_{E[\widetilde{A}]}(\epsilon \le x) \\ &= \begin{cases} 0, & x \le \gamma \\ \dfrac{x - \frac{(e^r+e^l)(a-c^l)}{2}}{4(a - c^l + \frac{(e^r+e^l)(a-c^l)}{2})}, & \gamma \le x \le a \\ \dfrac{c^r + \frac{(e^r+e^l)(c^r-a)}{2} + x - 2a}{4(c^r - a + \frac{(e^r+e^l)(c^r-a)}{2})}, & a \le x \le \zeta \\ 1, & \zeta \ge x \end{cases} \end{aligned} \quad (13)$$

where $\gamma = c^l - \frac{(e^r+e^l)(a-c^l)}{2}$ and $\zeta = c^r + \frac{(e^r+e^l)(c^r-a)}{2}$. We call A' the expected value of $\widetilde{A}$ and it is a range of T1 fuzzy set.

4. Formulation of a class of T2 fuzzy regression model

T1 fuzzy set's arithmetic operations have been studied by making use of the extension principle [41][42][43][44][45]. These studies have involved the definition of possibility. Tanaka and Watada figured out that fuzzy equations discussed by Sanchez can be regarded as a possibilistic structure in 1989. In the sequel, a possibilistic system has been applied to the linear regression analysis, A possibility structure has much advantage to deal with inputs and outputs with uncertainty. We will use the structure here as well. In our T2 fuzzy regression model, input data $\widetilde{X}_{ij}$ and output data $\widetilde{Y}_i$, for all $i = 1.\cdots, N$ and $k = 1, \cdots, M$ are assumed to be T2 fuzzy variables, which are defined as

$$\widetilde{Y}_i = (y_i, \mu_{Y_i}(y_i), \mu_{\widetilde{Y}_i}(y_i))\widetilde{X}_{ij} = (x_{ij}, \mu_{X_{ij}}(x_{ij}), \mu_{\widetilde{X}_{ij}}(x_{ij}))(14)$$

respectively.

y_i and x_{ij} are the crisp value; $\mu_{Y_i}(y_i)$ and $\mu_{X_{ij}}(x_{ij})$ are primary membership grades for y_i and x_{ij}; $\mu_{\widetilde{Y}_i}(y_i)$ and $\mu_{\widetilde{X}_{ij}}(x_{ij})$ are secondary membership grades for $\mu_{Y_i}(y_i)$ and $\mu_{X_{ij}}(x_{ij})$. These three factors construct the basis for a T2 fuzzy variable. Where i denotes sample i for $i = 1, \cdots, N$; j denotes for the jth attributes for $j = 1, 2, \cdots, M$.

As discussed before, we will use a possibilistic structure here. Let us denote fuzzy linear regression model with T1 fuzzy coefficients $A_1, \cdots, A_M$. Then the T2 fuzzy regression is in the form as follows:

$$\widetilde{Y}_i = A_1\widetilde{X}_{i1} + A_2\widetilde{X}_{i2} + \cdots + A_M\widetilde{X}_{iM}, \cdots\cdots\cdots\cdots\cdots (15)$$

where $\widetilde{Y}_i$ denotes an estimate of the T2 fuzzy output and $A_j = \left(\frac{A_j^l+A_j^r}{2}, A_j^l, A_j^r\right)_T$ are symmetric triangular fuzzy coefficients when triangular T2 fuzzy data $\widetilde{X_{ij}}$ are given for $i = 1, \cdots, N$ and $j = 1, \cdots, M$.

When outputs and inputs are defined as crisp value or T1 fuzzy variables, it is easy to determine the linear regression model's parameters by satisfying the estimated model includes all given outputs. We will mimic this process to formulate the constraints for T2 fuzzy regression model.

$$\begin{aligned} &\widetilde{Y}_i = A_1\widetilde{X}_{i1} + A_2\widetilde{X}_{i2} + \cdots + A_M\widetilde{X}_{iM} \underset{FR}{\supset} \widetilde{Y}_i, \\ &\qquad i = 1, \ldots N, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (16) \end{aligned}$$

where $\underset{FR}{\supset}$ is a T2 fuzzy inclusion relation whose precise meaning will be explained later on.

5. Building an expected value T2 fuzzy regression model based on creditability theory

We have defined the structure of a class of T2 fuzzy regression model above. In our definition, the inputs and outputs are all T2 fuzzy variables. And coefficients are T1 fuzzy sets.

In order to build an expected value T2 fuzzy regression model, we may reform an expected ary membership function for inputs $\widetilde{X}_v(ij)$. Thus, the $\widetilde{X}_v(ij)$ transfer to a range represented as $[E[\widetilde{X}_v(ij)^C] - E[\widetilde{X}_v(ij)^B], E[\widetilde{X}_v(ij)^C] + E[\widetilde{X}_v(ij)^B]]$. And it is the same to the outputs. We assume the inputs and outputs are all symmetric triangular T2 fuzzy variables whose primary membership functions and secondary ones are all in triangular form with a center C and two equal distance boundaries B. To simplify, we will use $[e_{\widetilde{X}_v(ij)}, \sigma_{\widetilde{X}_v(ij)}]$ instead of original the expression and $[e_{\widetilde{Y}_v(i)}, \sigma_{\widetilde{Y}_v(i)}]$ instead of $Y_{E[\widetilde{Y}_v(i)]}$.

Thus the T2 fuzzy regression model will reformulate as follows

[T2 fuzzy expected value regression model]

$$\begin{aligned} &\min_{A} \quad J(A) = \sum_{j=1}^{M} \left(A_j^r - A_j^l\right) \\ &\text{subject to} \\ &\qquad A_j^r \ge A_j^l, \\ &\qquad Y_i = \sum_{j=1}^{M} A_j \cdot [e_{\widetilde{X}_v(ij)}, \sigma_{\widetilde{X}_v(ij)}] \underset{h}{\supset} [e_{\widetilde{Y}_v(i)}, \sigma_{\widetilde{Y}_v(i)}], \\ &\qquad \text{for} \quad i = 1, \ldots N, j = 1, \cdots, M. \end{aligned} \quad (17)$$

As shown from the equation above, original T2 fuzzy inputs and outputs are replaced by their expected values, which are T1 fuzzy sets. Thus, Calculations between three dimensional T2 fuzzy sets are thus transferred to two-dimensional range calculations between T1 fuzzy sets. On the one hand, range calculations are much easier than three-dimensional ones. On the other hand, information is transformed from vertical style into range style without much distortion. However, we still have many constraints to meet. So we offer a easy solution as follows.

6. A solution based on heuristic method

Given we are not able to decide whether the coefficient is positive or negative at first. We may introduce a trial and error method to approach the consequence. The basic idea behind it is to eliminate the error of estimation on the polar of coefficient by checking the consistency of it. Once the polar of each coefficient becomes consistent, then we will take the outcome as the consequence. Besides, we may introduce a attribute L to accommodate the accuracy of the consequence Consider we have got the expected value of input $I_{ij} = [e_{\widetilde{X}_v(ij)} - \sigma_{\widetilde{X}_v(ij)}, e_{\widetilde{X}_v(ij)} + \sigma_{\widetilde{X}_v(ij)}]$

We may define: $I^C_{ij} = e_{\widetilde{X}_v(ij)} + \dfrac{\sigma_{\widetilde{X}_v(ij)} - \sigma_{\widetilde{X}_v(ij)}}{2}$ as the approximately estimation of the membership degree we will take I^C_{ij} as the first input to the model to roughly determine whether the coefficient is positive or not. In the constraints, we make all values included in the upper and lower boundary.

LP problem is described as follows:

$$\min_A \qquad J(A) = \sum_{j=1}^{M} \left(A^r_j - A^l_j \right)$$

subject to

$$\begin{array}{ll}
 & A^r_j \geq A^l_j, \\
(1) \rightarrow & Y_i = A_1 \cdot I^L_{i1} + A.I^L_i + \cdots \\
 & +A_M \cdot I^L_{iM} \underset{\sim}{\supseteq} [e_{\widetilde{Y}_v(i)}, \sigma_{\widetilde{Y}_v(i)}], \\
(2) \rightarrow & Y_i = A_1 \cdot I^U_{i1} + A_2 \cdot I^L_{i2} + \cdots \\
 & +A_M \cdot I^L_{iM} \underset{\sim}{\supseteq} [e_{\widetilde{Y}_v(i)}, \sigma_{\widetilde{Y}_v(i)}], \\
(3) \rightarrow & Y_i = A_1 \cdot I^L_{i1} + A_2 \cdot I^U_{i2} + \cdots \\
 & +A_M \cdot I^L_{iM} \underset{\sim}{\supseteq} [e_{\widetilde{Y}_v(i)}, \sigma_{\widetilde{Y}_v(i)}], \\
\vdots & \qquad \vdots \qquad\qquad \vdots \\
(2^M) \rightarrow & Y_i = A_1 \cdot I^U_{i1} + A_2 \cdot I^U_{i2} + \cdots \\
 & +A_K \cdot I^U_{iM} \underset{\sim}{\supseteq} [e_{\widetilde{Y}_v(i)}, \sigma_{\widetilde{Y}_v(i)}], \\
 & \text{for} \quad i = 1, \ldots N, j = 1, \cdots, M
\end{array} \quad \cdots\cdots (18)$$

Through solving the LP problem, we will get the first round's $A^r_j(1)$ and $A^l_j(1)$. At this time, $L = 1$, which means the first round's coefficients and shows in the way as $A_j(1)$. After got the first round's result, we can decide the form of consequnce of multiple of two ranges:

Consider a jth-coefficient if $A^r_j(1) \geq A^l_j(1) \geq 0$, we will assign $A_j(2) \cdot [e_{\widetilde{X}_v(ij)}, \sigma_{[\widetilde{X}_v(ij)]}]$ as

$A_j(2) \cdot [e_{\widetilde{X}_v(ij)}, \sigma_{[\widetilde{X}_v(ij)]}]$
$= [A^l_j(2) \cdot (e_{\widetilde{X}_v(ij)} - \sigma_{\widetilde{X}_v(ij)}), A^r_j(2) \cdot (e_{\widetilde{X}_v(ij)} + \sigma_{\widetilde{X}_v(ij)})]$

A more general case is as follows:

$(1) A^r_j(k) \geq A^l_j(k) \geq 0$
$A_j(k) \cdot [e_{\widetilde{X}_v(ij)}, \sigma_{[\widetilde{X}_v(ij)]}]$
$= [A^l_j(k) \cdot (e_{\widetilde{X}_v(ij)} - \sigma_{\widetilde{X}_v(ij)}), A^r_j(k) \cdot (e_{\widetilde{X}_v(ij)} + \sigma_{\widetilde{X}_v(ij)})]$
$(2) A^r_j(k) \leq A^l_j(k) \leq 0$:
$A_j(k) \cdot [e_{\widetilde{X}_v(ij)}, \sigma_{[\widetilde{X}_v(ij)]}]$
$= [A^l_j(k) \cdot (e_{\widetilde{X}_v(ij)} + \sigma_{\widetilde{X}_v(ij)}), A^r_j(k) \cdot (e_{\widetilde{X}_v(ij)} - \sigma_{\widetilde{X}_v(ij)})]$
$(3) A^l_j(k) \leq 0 \leq A^r_j(k)$:
$A_j(k) \cdot [e_{\widetilde{X}_v(ij)}, \sigma_{[\widetilde{X}_v(ij)]}]$
$= [A^l_j(k) \cdot (e_{\widetilde{X}_v(ij)} + \sigma_{\widetilde{X}_v(ij)}), A^r_j(k) \cdot (e_{\widetilde{X}_v(ij)} + \sigma_{\widetilde{X}_v(ij)})]$

By doing so we have formulate another LP problem ,which is the second round. We have assumed $A^r_j(1) \geq A^l_j(1) \geq 0$, so we can use the same form of LP problem above. After solving the LP problem above we may get the value of $A^r_j(2)$ and $A^l_j(2)$. At this time, $L = 2$.

If it satisfies

$$\begin{array}{l} A^l_j(1) \times A^l_j(2) \geq 0 \\ A^r_j(1) \times A^r_j(2) \geq 0 \end{array} \quad \cdots\cdots (19)$$

Then, it is possible for us to judge that the polar of coefficients has become consistent. Hence, we may take $A^r_j(2)$ and $A^r_j(2)$ as the final outcome in this case. If the condition is not satisfied, the procedure of iteration will be continued till we get both

$$\begin{array}{l} A^l_j(L-1) \times A^l_j(L) \geq 0 \\ A^r_j(L-1) \times A^r_j(L) \geq 0 \end{array} \quad \cdots\cdots (20)$$

Besides, we will assign a L0 (which is set as description of the accuracy for the problem at very beginning. After the polar has become consistent , we are allowed to repeat the procedure of iteration till the outcome meets our requirement of accuracy.

7. Application on Arbitrage Pricing Theory

In the section, we expand a famous pricing theory in finance, which is the arbitrage pricing theory(APT). We use T2F regression model based on creditability theory to redefine the formula of APT and to show the benefit of the new model in practice.

7.1 Mathmetical model of Arbitrage Pricing Theory

In finance, arbitrage pricing theory (APT) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various macro-economic factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient.

The form of arbitrage pricing theory is like:

$$E(r_j) = r_f + b_{j1} * RP_1 + b_{j2} * RP_2 + ...b_{jn} * RP_n \quad \cdots\cdots (22)$$

where $E(r_j)$ is the expected return of the jth asset; RP_k is the risk premium of the kth factor; r_f is the risk-free rate. That is, the expected return of an asset j is a linear function of the asset's sensitivities to the n factors. Note that there are some assumptions and requirements that have to be fulfilled for the latter to be correct: There must be perfect competition in the market, and the total number of factors may never surpass the total number of assets (in order to avoid the problem of matrix singularity),

As with the CAPM(another famous pricing theory), the factor-specific betas are found via a linear regression of historical security returns on the factor in question. Unlike the CAPM, the APT, however, does not itself reveal the identity of its priced factors - the number and nature of these factors is likely to change over time and between economies. As a result, this issue is essentially empirical in nature.

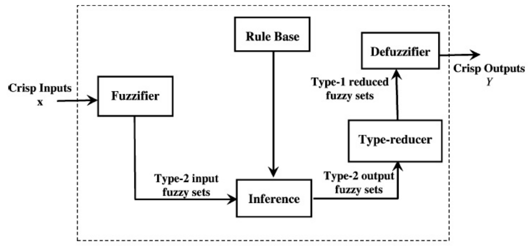

Fig. 1. The Architecture of a Type-2 Fuzzy Logic System

7.2 Modeling Background There are two main financial markets in China: the over-the-counter(called OTC) market and the exchange market. Price of assets in the OTC market is in the form of quotations, which is discrete and subjective based on offered or taken will from traders. Specifically speaking, for price index like the Shanghai interbank offered rate(called Shibor) or valuation for credit bond(called CBPV), quotation mechanism is usually as follows: the majority banks offered quotations according to their estimation of the future trend of financial markets. The market regulator collects all those quotations, then calculates a rough result as the current price. The price is supposed to contain all majority banks' opinions of the market and will be the basis for valuations of all other financial assets. Moreover, regulators will judge the quotation performance of each bank by some standards(called evaluation system) and give the evaluation to each quoter from time to time. The evaluation is supposed to restrict behaviors of quoters and exert effects on their quotations.

Considering that connections between different financial instruments become more and more obvious, there emerges study on linked movements of assets price recently, especially for bond market and money market. However, the quotation price of them is hard to quantify and model given its hybrid layers of uncertainties as mentioned above, which are the quotations range and regulators evaluation. Here, we use T2 fuzzy set to model them. More specifically, we use primary membership grade to describe how the quotation belonging to the current price series and use secondary membership grade to model those impact from regulators evaluations here. According to the meaning of APT, we select the whole model structure as a regression one.

7.3 Collecting of data We select both short term/high rank bonds yield to maturity(called YTM) and long term/high rank YTM as inputs like 1 years AAA YTM and 10 years AAA YTM to reflect basic conduction mechanism . Also we select low rank/short term bonds YTM as input like 1 years AA YTM to reflect conduction of risk preference. We choose 3 months Shibor as the representation of money surface in market and it is the output. According to the practical experience, 3MShibor is the best to express the price performance for money market. As mentioned above, price of Shibor and CBPV for one day is offered by majority banks as shown in table 1. In our case, we collect 2014 whole years CBPV from database of national association of financial market institutional investors(called Nafmii), which is a self-regulatory organization aiming to promote sustainable development of China OTC market and 2014 whole years Shibor from database of China foreign exchange trade center(called CFETS). Since CBPV is quoted twice a week and Shibor is quoted 3 times a week, we collect data at frequency of twice a week. Therefore, we have 104 sets of data. We choose 2014 to build our model because there is much fluctuation in financial markets in China thus is not proper to experiment the regular relationship between bond and money market.

As we can see from table 2, Nafmii evaluates the performance of each bank on credit debt spot rate quotation from respects such as quotation promptness, quotation effectiveness, deviation of transaction rate to quoted rate, etc. CFETS evaluate the performance of each bank on Shibor quotation from respects deviation of borrowing quotation, deviation of lending quotation, measurement of quotation error,etc.

After considering respects mentioned above, the regulators expertise group will grade each option then give a final assessment for accuracy of quotation such as always, often, sometimes, frequently, seldom to measure the performance of each bank. The evaluation is got from inside of Nafmii and CFETs and cannot used without authorization. We quantify the fuzziness of the evaluation of accuracy of quotation as always, 0.9,often,0.5,sometimes, 0.3,frequently, 0.7 and seldom, 0.1. After that, the quotation can be shown in table 4.

7.3.1 Processing of Data In order to quantify the quotation price sufficiently, we both take the offered price and evaluation from regulator into consideration. Given 2014-3-17 s quotations in the table as an example, the range of quotations of Shibor is from 5.25% to 5.59% and the final price given by regulator is 5.50%. We assume the final price given by regular has the primary membership as 1 and price out of the quotation range has the membership grade as 0. We use triangular membership to model the Shibor fuzzy set. The same is to CBPV. Because there are many banks quote besides 20 main banks and is not shown in the database of CEFTS. We assume that the quotations is continuous for the whole market.

[Custruction of T1 fuzzy set] Assumption 1: we use triangular function as the structure of T1 fuzzy set . The range constructs the base of the set and the final price given by regulator is the top of the triangle. According to the assumption: For 2014-3-17 s quotation, The Shibor is $[5.25\%, 5.50\%, 5.59\%]$; The 1y AAA is $[5.23\%, 5.73\%, 5.86\%]$; The 10y AAA is $[5.88\%, 6.23\%, 6.48\%]$, which is shown in figure 2.

Moreover, we take evaluation of the performance of each bank from regulator into consideration, which affects the secondary membership grade. In order to keep consistence, we assume T2 membership function is in the form of triangle as well. The evaluation factor affects the boundary and base of the secondary membership function.

[Custruction of T2 fuzzy set] Assumption2: We use s1 to represent the primary grade. The T2 membership function is represented as F_2. The T2 fuzziness is s2. We assume $F_2(s1)$ is always the center of the T2 fuzzy triangle and equals 1. The left boundary is defined as $max\{s1 - s2, 0\}$. The right boundary is defined as $min\{s1 + s2, 1\}$.

Take 2014-3-17s shibor quotation as an example, the primary membership grade of the quotation 5.45% is 0.8. We

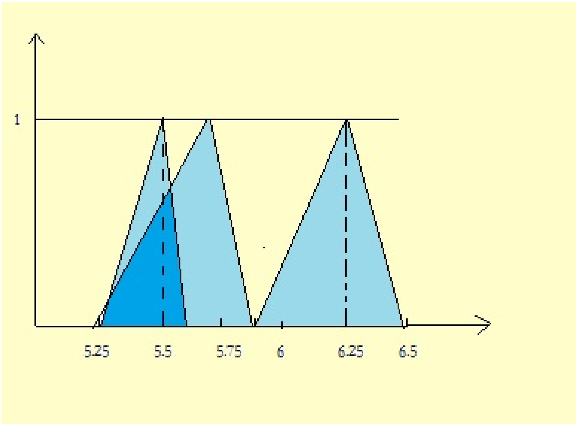

Fig. 2. The T1 fuzzy set of the quotations of money and bond market for 2014-3-17

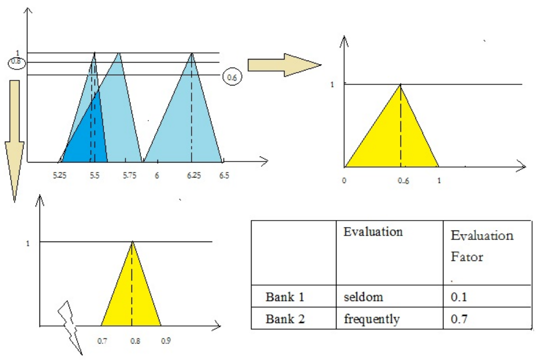

Fig. 3. The T2 fuzzy set of the quotations of money and bond market for 2014-3-17

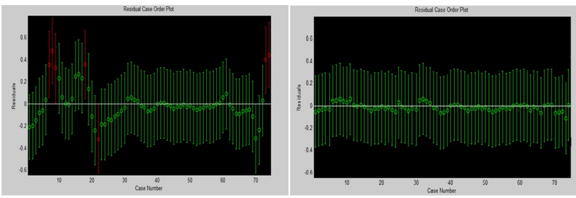

Fig. 4. The residual analysis traditional regression model and T2 fuzzy expected regression model

assume 0.8 is also the center of secondary membership grade. The 5.45% is quoted from a certain bank and its valuation of regulator is seldom, which means it gives bad quotations as usual. From the sense, we take evaluation factor 0.1 to construct its secondary boundaries and base, which is [0.7 ,0.8, 0.9]. Another example, the quotation 5.3% has the primary membership grade as 0.6. But its evaluation is frequently of which the factor is 0.7. Still 0 and 1 is the natural limitations. Thus its secondary membership is in the form of [0, 0.6, 1], which is shown in figure 3.

7.4 Formulation of the regression model The expected value of T2 fuzzy variable $\widetilde{A}_v$ is defined as follows.

$$E[\xi] = \int_{\Omega}\left[\int_0^{\infty} Cr\{\xi(\omega) \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi(\omega) \leq r\}dr\right]Pr(d\omega) \quad (23)$$

Table 1. 2015-3-28's quotation

Rank	Shibor 3M quoted price	1-year AAA quoted price	1-year AA quoted price	10-year AAA quoted price
Bank 1	5.50%	5.86%	6.48%	6.25%
Bank 2	5.50%	5.76%	6.47%	6.25%
Bank 3	5.49%	5.74%	6.47%	6.25%
Bank 4	5.50%	5.74%	6.47%	6.25%
Bank 5	5.50%	5.74%	6.46%	6.25%
Bank 6	5.50%	5.74%	6.46%	6.24%
Bank 7	5.50%	5.74%	6.46%	6.24%
Bank 8	5.25%	5.74%	6.46%	6.23%
Bank 9	5.50%	5.73%	6.46%	6.23%
Bank 10	5.50%	5.73%	6.46%	6.23%
Bank 11	5.51%	5.73%	6.46%	6.23%
Bank 12	5.50%	5.73%	6.46%	6.23%
Bank 13	5.50%	5.73%	6.45%	6.23%
Bank 14	5.59%	5.73%	6.45%	6.23%
Bank 15	5.52%	5.73%	6.45%	6.22%
Bank 16	5.45%	5.71%	6.43%	6.22%
Bank 17	5.49%	5.71%	6.42%	6.21%
Bank 18	2.49%	5.70%	6.40%	6.21%
Bank 19	2.49%	5.64%	6.35%	6.21%
Bank 20	2.51%	5.23%	5.88%	4.72%

Table 2. Evaluation system of quotation

Evaluation of Bond Spot Rate Quotation	Evaluation of Shibor Quotation
Quotation promptness(B1)	Deviation of Borrowing Rate Quotation(M1)
Quotation effectiveness(B2)	Deviation of Lending Rate Quotation(M2)
Deviation of transaction rate to Quoted rate(B3)	Measurement of Quotation Error(M3)
Measurement of Quotation Error(B4)	Stability of bilateral spread(M4)
Investors assessment(B5)	Trading volume on Quoted rate(M5)
Market Supervisors assessment(B6)	Quotation promptness(M6)
Issuers assessment(B7)	Quotation effectiveness(M7)

Table 3. Comparisons of performance between T2 fuzzy expected regression model, T1 fuzzy regression model and Regression model

	Coefficients	Prediction accuracy
T2-FER	[-2.25,-1.20];[1.75, 2.75];[0.0410,0.0705]	92%
T1-FR	[-2.45,-1.00];[1.50, 3.00];[0.030,0.0825]	89%
Regression	-1.68;2.25;0.06	54.5%

As defined in the former passage, we calculate the T2 expected value range from the original T2 fuzzy data range as shown in table 5. Actually, after transform the T2F set into T1F set, the new boundaries are loosing thus larger than original ones. It is because the new reduced T1F set contains more information than the original T1F set. We may calculate the new expected value of $\widetilde{A}_0$ is $A' = (c^l - \frac{(e^r+e^l)(a-c^l)}{2}, c^r + \frac{(e^r+e^l)(c^r-a)}{2})\Gamma_1$. Then, we expanded the traditional APT into the T2F APT as follows:

$$\widetilde{E(r_j)} = r_f + A_1\widetilde{RP_1} + A_2\widetilde{RP_2} + \cdots + A_M\widetilde{RP_M} \cdots \quad (24)$$

where coefficients A in the formula are T1F sets. The expected return of the jth asset $E(r_j)$ is in the form of T2 fuzzy sets. The risk premium of the kth factor RP_k is T2 fuzzy sets as well. Then we use T2 fuzzy expected value regression model(called T2-FER) based on the structure of APTto structure the data.

The result is shown as follows:

$Shibor_3M = [-2.25, -1.20]AAA_{1Y} + [1.75, 2.75]AA_{1Y} + [0.0410, 0.0705]AAA_{10Y}$

Table 4. 2015-6-1's quotation

Rank	Shibor 3M quoted price	1-year AAA quoted price	1-year AA quoted price	10-year AAA quoted price	Evaluation
Bank 1	5.50%	5.86%	6.48%	6.25%	Always
Bank 2	5.50%	5.76%	6.47%	6.25%	Sometimes
Bank 3	5.49%	5.74%	6.47%	6.25%	Seldom
Bank 4	5.50%	5.74%	6.47%	6.25%	Often
Bank 5	5.50%	5.74%	6.46%	6.25%	Always
Bank 6	5.50%	5.74%	6.46%	6.24%	Frequently
Bank 7	5.50%	5.74%	6.46%	6.24%	Often
Bank 8	5.25%	5.74%	6.46%	6.23%	Often
Bank 9	5.50%	5.73%	6.46%	6.23%	Often
Bank 10	5.50%	5.73%	6.46%	6.23%	Sometimes
Bank 11	5.51%	5.73%	6.46%	6.23%	Frequently
Bank 12	5.50%	5.73%	6.46%	6.23%	Frequently
Bank 13	5.50%	5.73%	6.45%	6.23%	Always
Bank 14	5.59%	5.73%	6.45%	6.23%	Always
Bank 15	5.52%	5.73%	6.45%	6.22%	Alway
Bank 16	5.45%	5.71%	6.43%	6.22%	Often
Bank 17	5.49%	5.71%	6.42%	6.21%	Often
Bank 18	2.49%	5.70%	6.40%	6.21%	Often
Bank 19	2.49%	5.64%	6.35%	6.21%	Often
Bank 20	2.51%	5.23%	5.88%	4.72%	Frequently

Table 5. Input-output data with confidence interval

Sample	Output	Inputs		
i	$I[E^L_Y, E^R_Y]$	$I[E^L_{X_1}, E^R_{X_1}]$	$\cdots$	$I[E^L_{X_K}, E^R_{X_K}]$
1	$I[E^L_{Y_1}, E^R_{Y_1}]$	$I[E^L_{X_{11}}, E^R_{X_{11}}]$	$\cdots$	$I[E^L_{X_{1K}}, E^R_{X_{1K}}]$
2	$I[E^L_{Y_2}, E^R_{Y_2}]$	$I[E^L_{X_{21}}, E^R_{X_{21}}]$	$\cdots$	$I[E^L_{X_{2K}}, E^R_{X_{2K}}]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$I[E^L_{Y_i}, E^R_{Y_i}]$	$I[E^L_{X_{i1}}, E^R_{X_{i1}}]$	$\cdots$	$I[E^L_{X_{iK}}, E^R_{X_{iK}}]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	$I[E^L_{Y_N}, E^R_{Y_N}]$	$I[E^L_{X_{N1}}, E^R_{X_{N1}}]$	$\cdots$	$I[E^L_{X_{NK}}, E^R_{X_{NK}}]$

After we build the T2-FER to the training data, we find out that the money surface is positively relative to the 10 years AAA credit bonds rate and much positively related to the 1 years AA credit bonds rate. However, the 1 years AAA bonds rate is negatively relative to Shibor. We try to explain it as follows: when money surface is loosen, investors will improve their risk appetite and put much money in high return asset such as low rank debt; 1 years AAA has much liquidity and it is to some extent the substitute of the lending of money, thus its price compete with Shibor 3M. According to macro economy theory, long term bonds rate indicates the situation of economy. Rise of it means economy is seeing a recover, which stimulates the price of all assets. Thus it has a weak positive linkage with money price.

7.5 Comparisons with traditional models Before comparisons with other method, we make some assumptions as follows: First, we assume the prediction of accuracy could be measured by overlap between the models output and the original value. For crisp value, if the output not equals the original data , we measure it as not correct. For fuzzy data, if the overlap between is not above 80%, then we say it is not accurate. We use matlab platform to implement T2-FER,T1-FR and crisp regression model. Considering that financial market has quarter effect, we use 2014 whole years data to train the model. And we use the data of first quarter of 2015 to test the model. To show the benefit of the new model, we also do the residual analysis. We use creditability theory to calculate the expected value of the residual of the T2-FER and is shown in the right part of figure 4. As we can see, the residual is much smooth than the traditional regression model. The prediction accuracy is shown in table 3. As we can see, T2-FER has a bit higher accuracy than T1-FR while the range of coefficients is much precise as shown in table 4. Also they both outperform than traditional method. Moreover, we use heuristic method to calculate the problem instead of traditional vertices method. In vertices method, there are 2K inequalities for each sample i. Therefore, we will have $K + 2 * N * 2K$ inequalities in total. Unfortunately this problem cannot be solved within a reasonable computing time when K becomes even moderately large. For example, when we have 10,000 features and 100,000 samples, the linear programming problem will come with $2 * 2^10,000$ constraints and 10,000 non-negative constraints. Heuristic method use irritation to try error s. The average irritation is less than 10 times for 100,000 samples thus can save most of calculations. There is evidence shown that heuristic method is much efficient.

8. Concluding Remarks

In this paper, we build a T2 fuzzy regression model based on creditability theory. The innovation of this paper stands on several stakes as follows: 1) we defined the expected value for T2 fuzzy variable based on creditability theory. 2) we formulated a T2 fuzzy regression model capable of dealing with T2 fuzzy inputs and outputs 3) Moreover, we offered an efficient heuristic solution. 4) We have expanded the traditional arbitrage pricing theory and build a new T2F APT. This paper generalized our previous work[22][23]. Our further work will try to define a new structure of T2F regression model with T2F coefficients and compare the effect of different defuzzification methods under the new structure.

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